Neutrino Theory: Review

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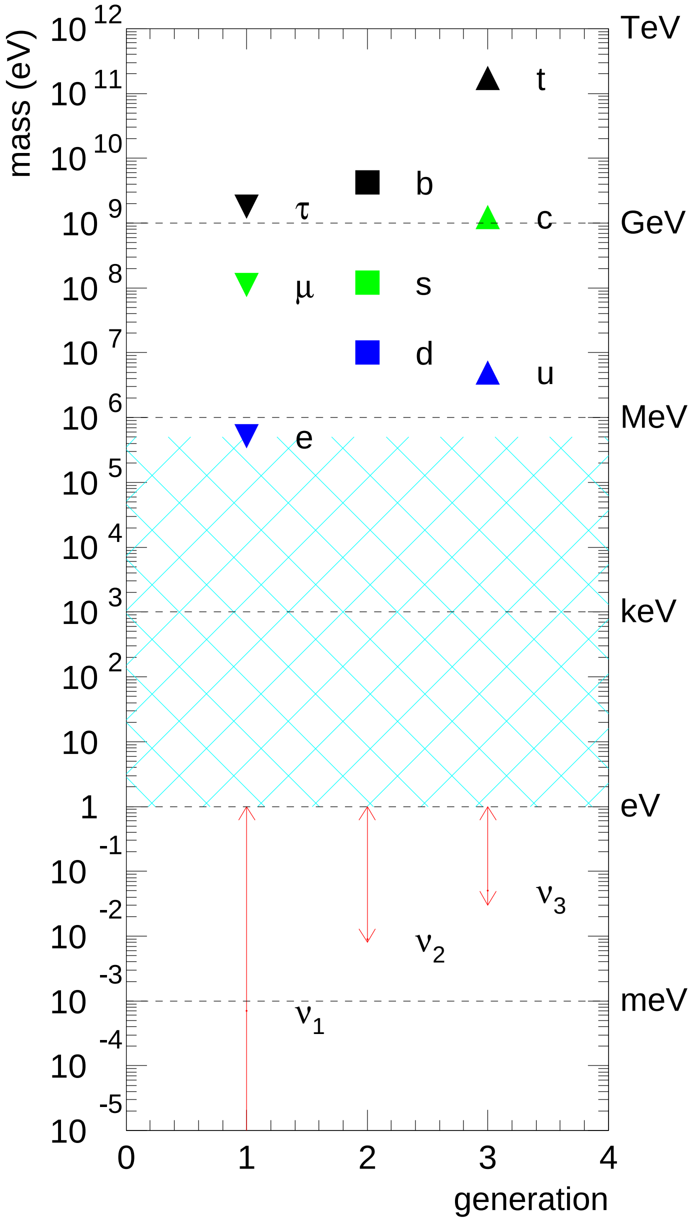
Outline

1. What We Are Trying to Understand;
2. Why Are Neutrino Masses Small?;
3. Quick Example – the Seesaw Mechanism: Three Avenues Toward Tiny Neutrino Masses, with Consequences;
4. A Fourth Avenue: Neutrino Masses from Involved $\Delta L = 2$ New Physics (Loops);
5. Comments on Neutrino Mixing.



What We Are Trying To Understand:

⇐ NEUTRINOS HAVE TINY MASSES

⇓ LEPTON MIXING IS "WEIRD" ⇓

$$V_{MNS} \sim \begin{pmatrix} 0.8 & 0.5 & \mathbf{0.2} \\ 0.4 & 0.6 & 0.7 \\ 0.4 & 0.6 & 0.7 \end{pmatrix} \qquad V_{CKM} \sim \begin{pmatrix} 1 & 0.2 & 0.001 \\ 0.2 & 1 & 0.01 \\ 0.001 & 0.01 & 1 \end{pmatrix}$$

$\boxed{\textbf{What Does It Mean?}}$

What is the New Standard Model? [νSM]

The short answer is – WE DON'T KNOW. Not enough available info!

$$\Updownarrow$$

Equivalently, there are several completely different ways of addressing neutrino masses. The key issue is to understand what else the νSM candidates can do. [are they falsifiable?, are they "simple"?, do they address other outstanding problems in physics?, etc]

We need more experimental input.

Candidate νSM: The One I'll Concentrate On

SM as an effective field theory – non-renormalizable operators

$$\mathcal{L}_{\nu\mathrm{SM}} \supset -y_{ij}\frac{L^i H L^j H}{2\Lambda} + \mathcal{O}\left(\frac{1}{\Lambda^2}\right) + H.c.$$

There is only one dimension five operator [Weinberg, 1979]. If mass of new physics $\gg 1$ TeV, it leads to only one observable consequence...

$$\text{after EWSB: } \mathcal{L}_{\nu\mathrm{SM}} \supset \frac{m_{ij}}{2}\nu^i\nu^j; \quad m_{ij} = y_{ij}\frac{v^2}{\Lambda}.$$

- Neutrino masses are small: $\Lambda \gg v \rightarrow m_\nu \ll m_f$ ($f = e, \mu, u, d$, etc)
- Neutrinos are Majorana fermions – Lepton number is violated!
- νSM effective theory – not valid for energies above *at most* Λ/y.
- Define $y_{\max} \equiv 1 \Rightarrow$ data require $\Lambda \sim 10^{14}$ GeV.

What else is this "good for"? Depends on the ultraviolet completion!

The Seesaw Lagrangian

A simple[a], renormalizable Lagrangian that allows for neutrino masses is

$$\mathcal{L}_\nu = \mathcal{L}_{\rm old} - \lambda_{\alpha i} L^\alpha H N^i - \sum_{i=1}^{3} \frac{M_i}{2} N^i N^i + H.c.,$$

where N_i ($i = 1, 2, 3$, for concreteness) are SM gauge singlet fermions. $\mathcal{L}_\nu$ is the most general, renormalizable Lagrangian consistent with the SM gauge group and particle content, plus the addition of the N_i fields.

After electroweak symmetry breaking, $\mathcal{L}_\nu$ describes, besides all other SM degrees of freedom, six Majorana fermions: **six neutrinos.**

[a]Only requires the introduction of three fermionic degrees of freedom, no new interactions or symmetries.

To be determined from data: λ and M.

The data can be summarized as follows: there is evidence for three neutrinos, mostly "active" (linear combinations of ν_e, ν_μ, and ν_τ). At least two of them are massive and, if there are other neutrinos, they have to be "sterile."

This provides very little information concerning the magnitude of M_i (assume $M_1 \sim M_2 \sim M_3$).

Theoretically, there is prejudice in favor of very large M: $M \gg v$. Popular examples include $M \sim M_{\rm GUT}$ (GUT scale), or $M \sim 1$ TeV (EWSB scale).

Furthermore, $\lambda \sim 1$ translates into $M \sim 10^{14}$ GeV, while thermal leptogenesis requires the lightest M_i to be around 10^{10} GeV.

we can impose very, very few experimental constraints on M

What We Know About M:

- $M = 0$: the six neutrinos "fuse" into three Dirac states. Neutrino mass matrix given by $\mu_{\alpha i} \equiv \lambda_{\alpha i} v$.

  The symmetry of $\mathcal{L}_\nu$ is enhanced: $U(1)_{B-L}$ is an exact global symmetry of the Lagrangian if all M_i vanish. Small M_i values are 'tHooft natural.

- $M \gg \mu$: the six neutrinos split up into three mostly active, light ones, and three, mostly sterile, heavy ones. The light neutrino mass matrix is given by $m_{\alpha\beta} = \sum_i \mu_{\alpha i} M_i^{-1} \mu_{\beta i}$ $[m \propto 1/\Lambda \ \Rightarrow \ \Lambda = M/\mu^2]$.

  This the **seesaw mechanism.** Neutrinos are Majorana fermions. Lepton number is not a good symmetry of $\mathcal{L}_\nu$, even though L-violating effects are hard to come by.

- $M \sim \mu$: six states have similar masses. Active–sterile mixing is very large. This scenario is (generically) ruled out by active neutrino data (atmospheric, solar, KamLAND, K2K, etc).

Why are Neutrino Masses Small in the $M \neq 0$ Case?

If $\mu \ll M$, below the mass scale M,

$$\mathcal{L}_5 = \frac{LHLH}{\Lambda}.$$

Neutrino masses are small if $\Lambda \gg \langle H \rangle$. Data require $\Lambda \sim 10^{14}$ GeV.

In the case of the seesaw,

$$\Lambda \sim \frac{M}{\lambda^2},$$

so neutrino masses are small if either

- they are generated by physics at a very high energy scale $M \gg v$ (high-energy seesaw); or
- they arise out of a very weak coupling between the SM and a new, hidden sector (low-energy seesaw); or
- cancellations among different contributions render neutrino masses accidentally small ("fine-tuning").

Low-Energy Seesaw [AdG PRD72,033005)]

The other end of the M spectrum ($M < 100$ GeV). What do we get?

- Neutrino masses are small because the Yukawa couplings are very small $\lambda \in [10^{-6}, 10^{-11}]$;
- No standard thermal leptogenesis – right-handed neutrinos way too light;
- No obvious connection with other energy scales (EWSB, GUTs, etc);
- Right-handed neutrinos are propagating degrees of freedom. They look like sterile neutrinos $\Rightarrow$ sterile neutrinos associated with the fact that the active neutrinos have mass;
- sterile–active mixing can be predicted – hypothesis is falsifiable!
- Small values of M are natural (in the 'tHooft sense). In fact, theoretically, no value of M should be discriminated against!

More Details, assuming three right-handed neutrinos N:

$$m_\nu = \begin{pmatrix} 0 & \lambda v \\ (\lambda v)^t & M \end{pmatrix},$$

M is diagonal, and all its eigenvalues are real and positive. The charged lepton mass matrix also diagonal, real, and positive.

To leading order in $(\lambda v)M^{-1}$, the three lightest neutrino mass eigenvalues are given by the eigenvalues of

$$m_a = \lambda v M^{-1} (\lambda v)^t,$$

where m_a is the mostly active neutrino mass matrix, while the heavy sterile neutrino masses coincide with the eigenvalues of M.

6×6 mixing matrix U $[U^t m_\nu U = \text{diag}(m_1,m_2,m_3,m_4,m_5,m_6)]$ is

$$U=\left(\begin{array}{cc} V & \Theta \\ -\Theta^\dagger V & 1_{n\times n}\end{array}\right),$$

where V is the active neutrino mixing matrix (MNS matrix)

$$V^t m_a V = \text{diag}(m_1,m_2,m_3),$$

and the matrix that governs active–sterile mixing is

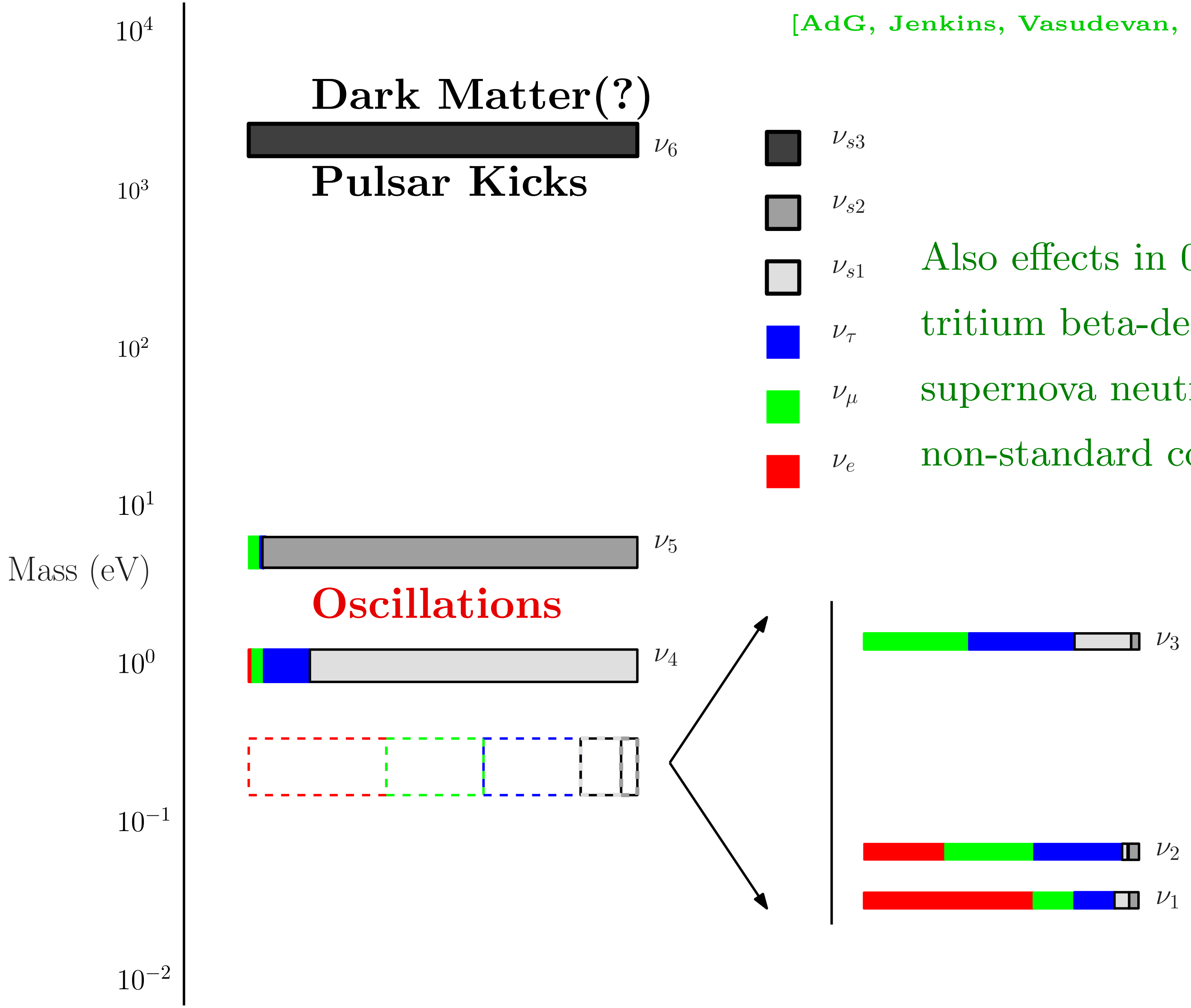
$$\Theta = (\lambda v)^* M^{-1}.$$

One can solve for the Yukawa couplings and re-express

$$\Theta = V\sqrt{\text{diag}(m_1,m_2,m_3)}R^\dagger M^{-1/2},$$

where R is a complex orthogonal matrix $RR^t=1$.

[AdG, Jenkins, Vasudevan, PRD75, 013003 (2007)]



Also effects in $0\nu\beta\beta$, tritium beta-decay, supernova neutrino oscillations, non-standard cosmology.

Predictions: **Neutrinoless Double-Beta Decay**

The exchange of Majorana neutrinos mediates lepton-number violating neutrinoless double-beta decay, $0\nu\beta\beta$: $Z \to (Z+2)e^-e^-$.

For light enough neutrinos, the amplitude for $0\nu\beta\beta$ is proportional to the effective neutrino mass

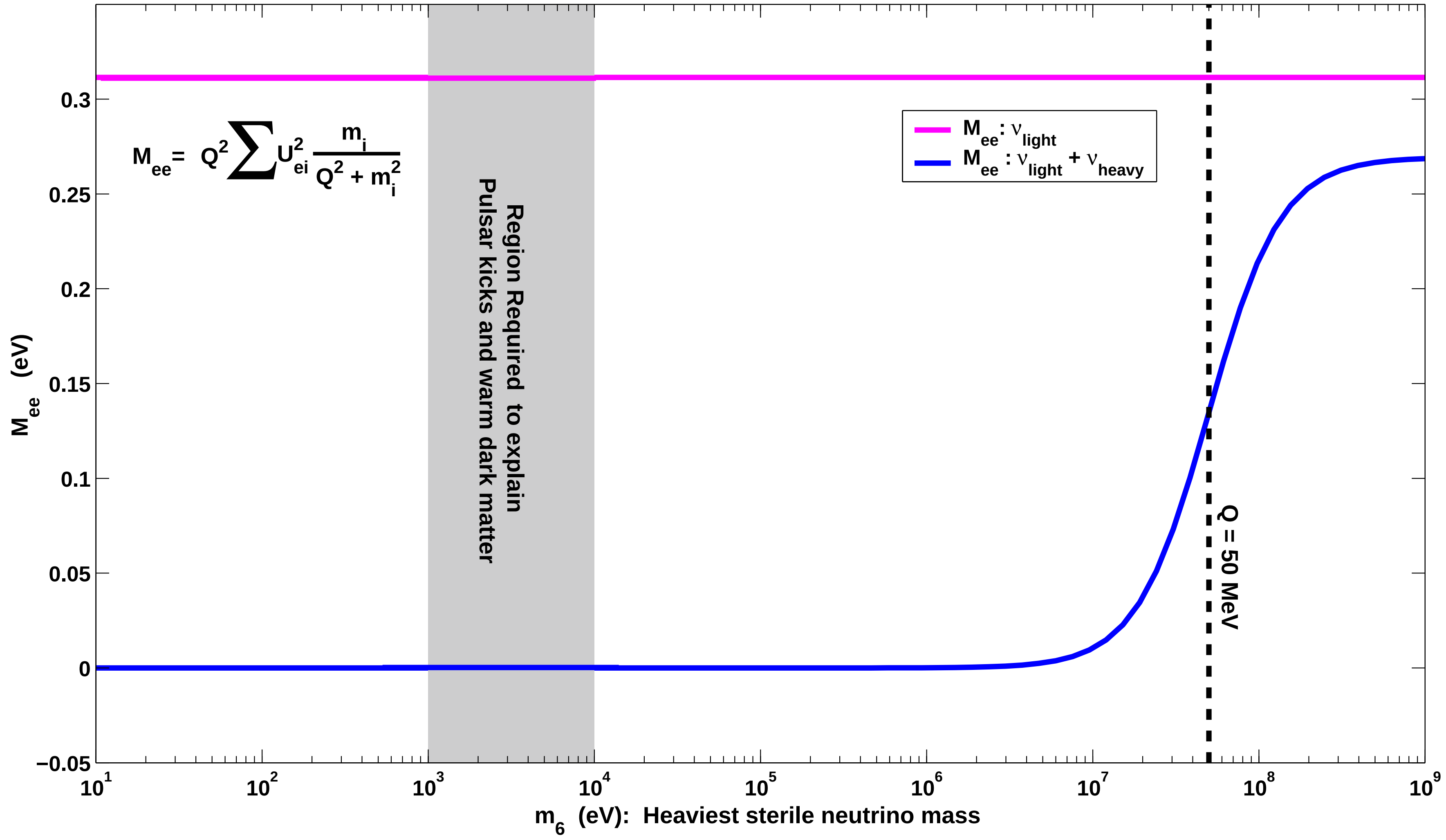
$$m_{ee} = \left|\sum_{i=1}^{6} U_{ei}^2 m_i\right| \sim \left|\sum_{i=1}^{3} U_{ei}^2 m_i + \sum_{i=1}^{3} \vartheta_{ei}^2 M_i\right|.$$

However, upon further examination, $m_{ee} = 0$ in the eV-seesaw. **The contribution of light and heavy neutrinos exactly cancels!** This seems to remain true to a good approximation as long as $M_i \ll 1$ MeV.

$$\left[\ \mathcal{M} = \begin{pmatrix} 0 & \mu^{\mathrm{T}} \\ \mu & M \end{pmatrix} \quad \to \quad m_{ee} \text{ is identically zero!}\ \right]$$

(lack of) sensitivity in $0\nu\beta\beta$ due to seesaw sterile neutrinos

[AdG, Jenkins, Vasudevan, hep-ph/0608147]



What if 1 GeV$< M <1$ TeV?

Naively, one expects

$$\Theta \sim \sqrt{\frac{m_a}{M}} < 10^{-5}\sqrt{\frac{1\text{ GeV}}{M}},$$

such that, for $M = 1$ GeV and above, sterile neutrino effects are mostly negligible.

However,

$$\Theta = V\sqrt{\mathrm{diag}(m_1, m_2, m_3)}R^{\dagger}M^{-1/2},$$

and the magnitude of the entries of R can be arbitrarily large [$\cos(ix) = \cosh x \gg 1$ if $x > 1$].

This is true as long as

- $\lambda v \ll M$ (seesaw approximation holds)
- $\lambda < 4\pi$ (theory is "well-defined")

This implies that, in principle, Θ is a quasi-free parameter – independent from light neutrino masses and mixing – as long as $\Theta \ll 1$ and $M < 1$ TeV.

What Does $R \gg 1$ Mean?

It is illustrative to consider the case of one active neutrino of mass m_3 and two sterile ones, and further assume that $M_1 = M_2 = M$. In this case,

$$\begin{aligned} \Theta &= \sqrt{\frac{m_3}{M}} \left(\begin{array}{cc} \cos\zeta & \sin\zeta \end{array} \right), \\ \lambda v &= \sqrt{m_3 M} \left(\begin{array}{cc} \cos\zeta^* & \sin\zeta^* \end{array} \right) \equiv \left(\begin{array}{cc} \lambda_1 & \lambda_2 \end{array} \right). \end{aligned}$$

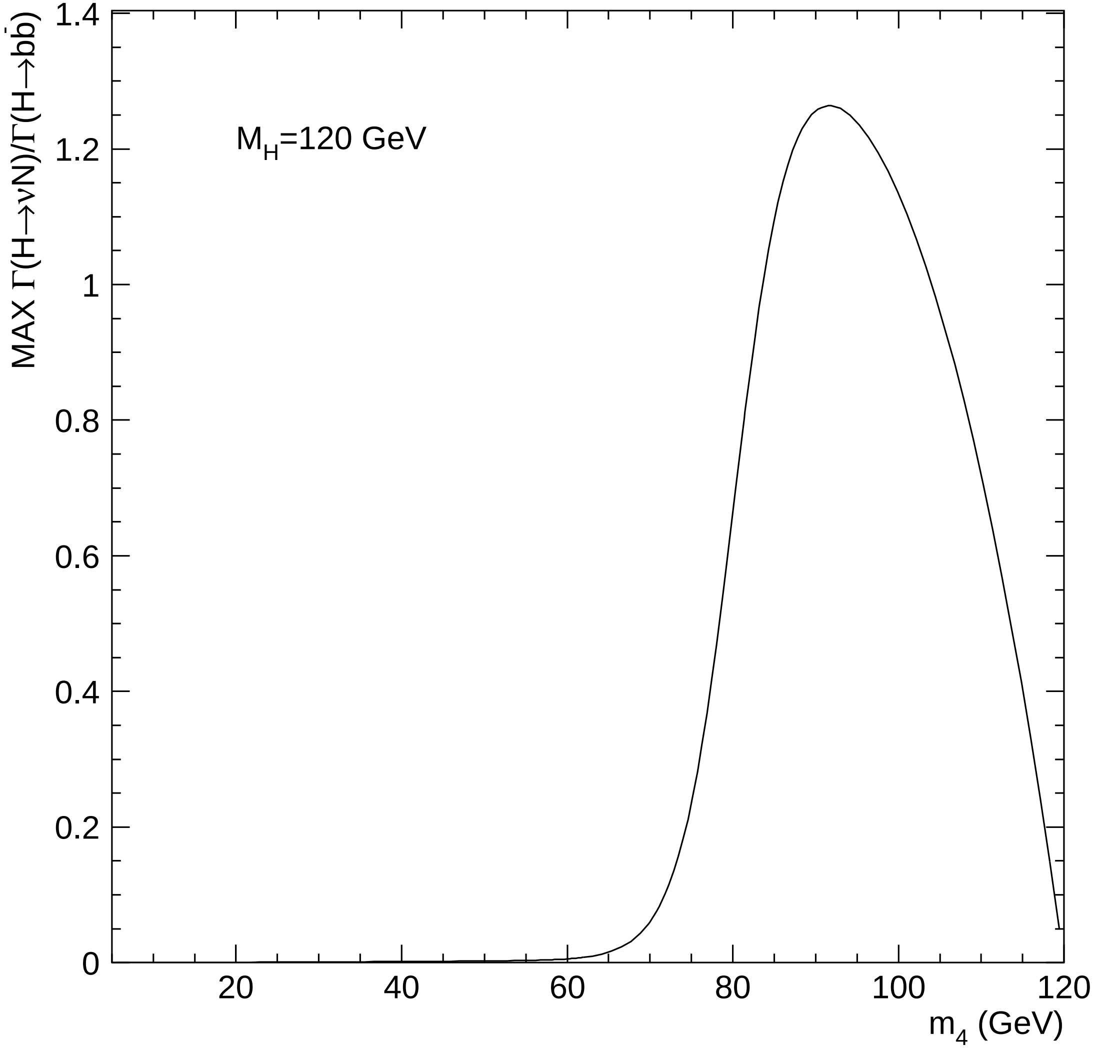
If ζ has a large imaginary part $\Rightarrow$ Θ is (exponentially) larger than $(m_3/M)^{1/2}$, λ_i neutrino Yukawa couplings are much larger than $\sqrt{m_3 M}/v$

The reason for this is a strong cancellation between the contribution of the two different Yukawa couplings to the active neutrino mass $\Rightarrow$ $m_3 = \lambda_1^2 v^2/M + \lambda_2^2 v^2/M$.

For example: $m_3 = 0.1$ eV, $M = 100$ GeV, $\zeta = 14i \Rightarrow$ $\lambda_1 \sim 0.244, \lambda_2 \sim -0.244i$, while $|y_1| - |y_2| \sim 3.38 \times 10^{-13}$.

Weak Scale Seesaw, and Accidentally Light Neutrino Masses

[AdG arXiv:0706.1732 [hep-ph]]



What does the seesaw Lagrangian predict for the LHC?

Nothing much, unless. . .

- $M_N \sim 1 - 100$ GeV,
- Yukawa couplings larger than naive expectations.

$\Leftarrow$ $H \to \nu N$ as likely as $H \to b\bar{b}$!

$\Big($NOTE: $N \to \ell q' \bar{q}$ or $\ell\ell'\nu$ (prompt)

"Weird" Higgs decay signature! $\Big)$

[+ Lepton Number Violation at Colliders]

Fourth Avenue: Higher Order Neutrino Masses from $\Delta L = 2$ Physics.

Imagine that there is new physics that breaks lepton number by 2 units at some energy scale Λ, but that it does not, in general, lead to neutrino masses at the tree level.

We know that neutrinos will get a mass at some order in perturbation theory – which order is model dependent!

For example:

- SUSY with trilinear R-parity violation – neutrino masses at one-loop;
- Zee model – neutrino masses at one-loop;
- Babu and Ma – neutrino masses at two loops;
- Chen, *et al.* 0706.1964 – neutrino masses at two loops;
- etc

AdG, Jenkins, 0708.1344 [hep-ph]

Effective Operator Approach

(there are 129 of them if you discount different Lorentz structures!)

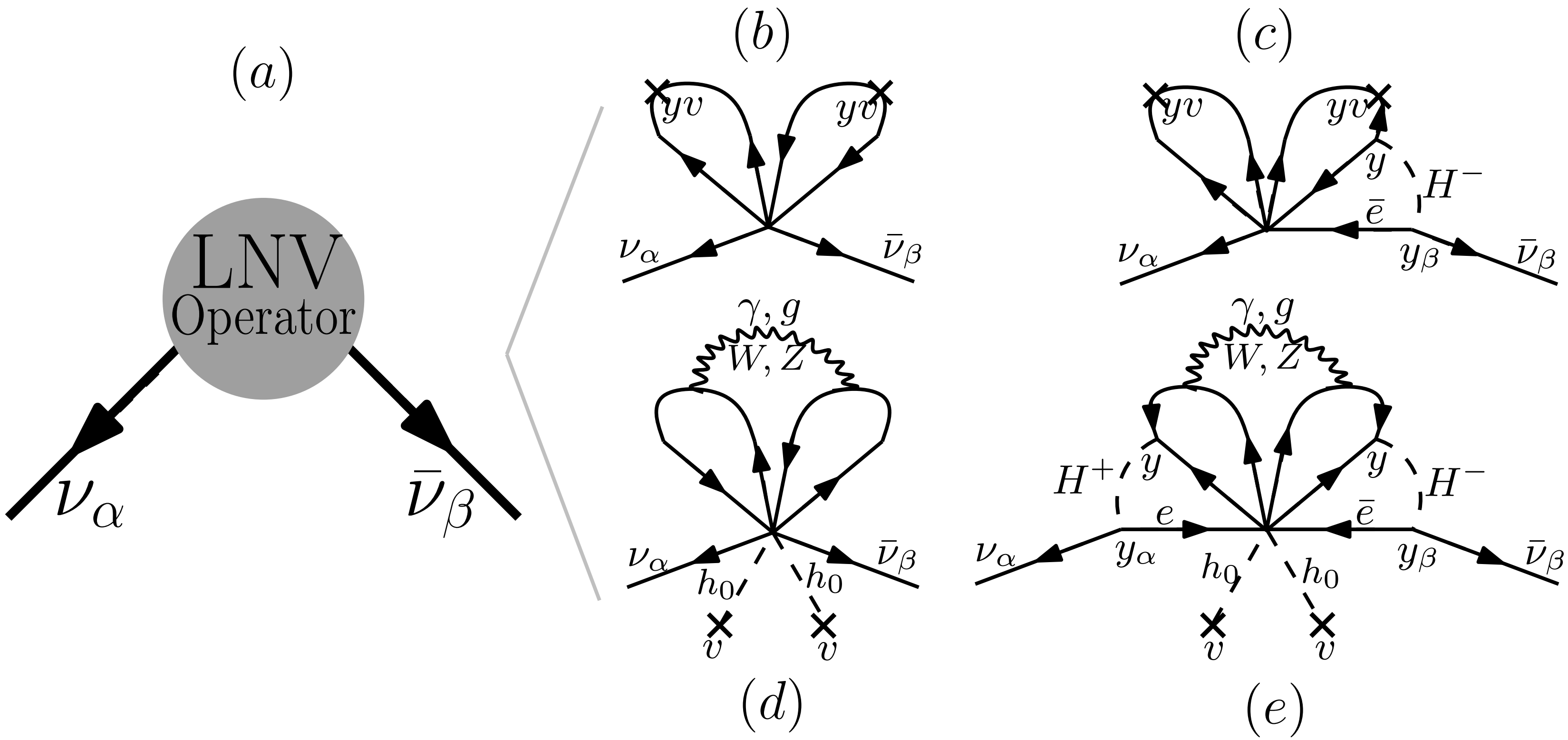
classified by Babu and Leung in NPB**619**,667(2001)

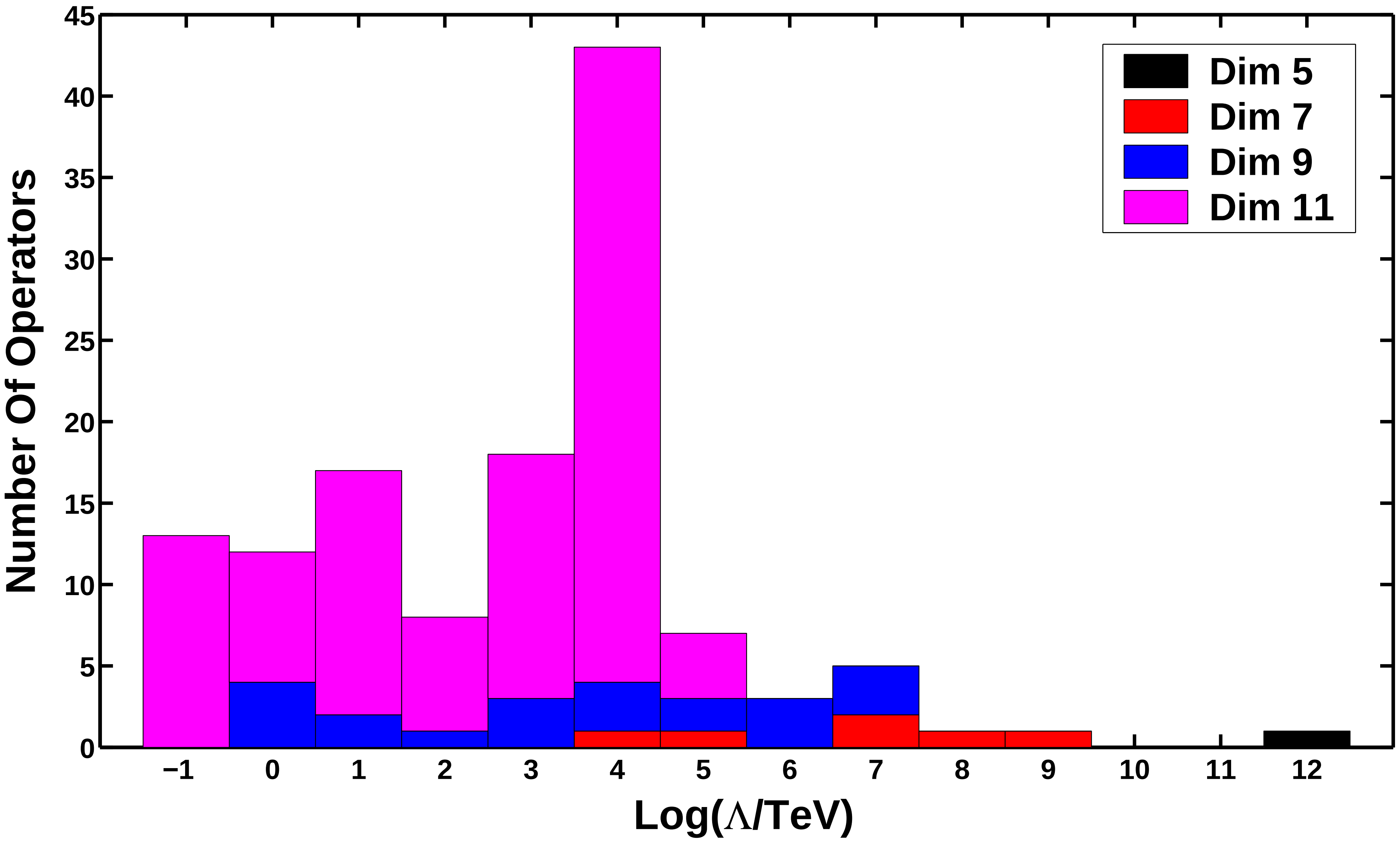
13	$L^iL^j\overline{Q}_i\overline{u^c}L^le^c\epsilon_{jl}$	$\frac{y_\ell y_u}{(16\pi^2)^2}\frac{v^2}{\Lambda}$	2×10^5	$\beta\beta0\nu$
14_a	$L^iL^j\overline{Q}_k\overline{u^c}Q^kd^c\epsilon_{ij}$	$\frac{y_d y_u g^2}{(16\pi^2)^3}\frac{v^2}{\Lambda}$	1×10^3	$\beta\beta0\nu$
14_b	$L^iL^j\overline{Q}_i\overline{u^c}Q^ld^c\epsilon_{jl}$	$\frac{y_d y_u}{(16\pi^2)^2}\frac{v^2}{\Lambda}$	6×10^5	$\beta\beta0\nu$
15	$L^iL^jL^kd^c\overline{L}_i\overline{u^c}\epsilon_{jk}$	$\frac{y_d y_u g^2}{(16\pi^2)^3}\frac{v^2}{\Lambda}$	1×10^3	$\beta\beta0\nu$
16	$L^iL^je^cd^c\overline{e^c}\overline{u^c}\epsilon_{ij}$	$\frac{y_d y_u g^4}{(16\pi^2)^4}\frac{v^2}{\Lambda}$	2	$\beta\beta0\nu$, LHC
17	$L^iL^jd^cd^c\overline{d^c}\overline{u^c}\epsilon_{ij}$	$\frac{y_d y_u g^4}{(16\pi^2)^4}\frac{v^2}{\Lambda}$	2	$\beta\beta0\nu$, LHC
18	$L^iL^jd^cu^c\overline{u^c}\overline{u^c}\epsilon_{ij}$	$\frac{y_d y_u g^4}{(16\pi^2)^4}\frac{v^2}{\Lambda}$	2	$\beta\beta0\nu$, LHC
19	$L^iQ^jd^cd^c\overline{e^c}\overline{u^c}\epsilon_{ij}$	$y_{\ell_\beta}\frac{y_d^2 y_u}{(16\pi^2)^3}\frac{v^2}{\Lambda}$	1	$\beta\beta0\nu$, HElnv, LHC, m
20	$L^id^c\overline{Q}_i\overline{u^c}\overline{e^c}\overline{u^c}$	$y_{\ell_\beta}\frac{y_d y_u^2}{(16\pi^2)^3}\frac{v^2}{\Lambda}$	40	$\beta\beta0\nu$, mix
21_a	$L^iL^jL^ke^cQ^lu^cH^mH^n\epsilon_{ij}\epsilon_{km}\epsilon_{ln}$	$\frac{y_\ell y_u}{(16\pi^2)^2}\frac{v^2}{\Lambda}\left(\frac{1}{16\pi^2}+\frac{v^2}{\Lambda^2}\right)$	2×10^3	$\beta\beta0\nu$
21_b	$L^iL^jL^ke^cQ^lu^cH^mH^n\epsilon_{il}\epsilon_{jm}\epsilon_{kn}$	$\frac{y_\ell y_u}{(16\pi^2)^2}\frac{v^2}{\Lambda}\left(\frac{1}{16\pi^2}+\frac{v^2}{\Lambda^2}\right)$	2×10^3	$\beta\beta0\nu$
22	$L^iL^jL^ke^c\overline{L}_k\overline{e^c}H^lH^m\epsilon_{il}\epsilon_{jm}$	$\frac{g^2}{(16\pi^2)^3}\frac{v^2}{\Lambda}$	4×10^4	$\beta\beta0\nu$
23	$L^iL^jL^ke^c\overline{Q}_k\overline{d^c}H^lH^m\epsilon_{il}\epsilon_{jm}$	$\frac{y_\ell y_d}{(16\pi^2)^2}\frac{v^2}{\Lambda}\left(\frac{1}{16\pi^2}+\frac{v^2}{\Lambda^2}\right)$	40	$\beta\beta0\nu$
24_a	$L^iL^jQ^kd^cQ^ld^cH^m\overline{H}_i\epsilon_{jk}\epsilon_{lm}$	$\frac{y_d^2}{(16\pi^2)^3}\frac{v^2}{\Lambda}$	1×10^2	$\beta\beta0\nu$
24_b	$L^iL^jQ^kd^cQ^ld^cH^m\overline{H}_i\epsilon_{jm}\epsilon_{kl}$	$\frac{y_d^2}{(16\pi^2)^3}\frac{v^2}{\Lambda}$	1×10^2	$\beta\beta0\nu$
25	$L^iL^jQ^kd^cQ^lu^cH^mH^n\epsilon_{im}\epsilon_{jn}\epsilon_{kl}$	$\frac{y_d y_u}{(16\pi^2)^2}\frac{v^2}{\Lambda}\left(\frac{1}{16\pi^2}+\frac{v^2}{\Lambda^2}\right)$	4×10^3	$\beta\beta0\nu$
26_a	$L^iL^jQ^kd^c\overline{L}_i\overline{e^c}H^lH^m\epsilon_{jl}\epsilon_{km}$	$\frac{y_\ell y_d}{(16\pi^2)^3}\frac{v^2}{\Lambda}$	40	$\beta\beta0\nu$
26_b	$L^iL^jQ^kd^c\overline{L}_k\overline{e^c}H^lH^m\epsilon_{il}\epsilon_{jm}$	$\frac{y_\ell y_d}{(16\pi^2)^2}\frac{v^2}{\Lambda}\left(\frac{1}{16\pi^2}+\frac{v^2}{\Lambda^2}\right)$	40	$\beta\beta0\nu$
27_a	$L^iL^jQ^kd^c\overline{Q}_i\overline{d^c}H^lH^m\epsilon_{jl}\epsilon_{km}$	$\frac{g^2}{(16\pi^2)^3}\frac{v^2}{\Lambda}$	4×10^4	$\beta\beta0\nu$
27_b	$L^iL^jQ^kd^c\overline{Q}_k\overline{d^c}H^lH^m\epsilon_{il}\epsilon_{jm}$	$\frac{g^2}{(16\pi^2)^3}\frac{v^2}{\Lambda}$	4×10^4	$\beta\beta0\nu$
28_a	$L^iL^jQ^kd^c\overline{Q}_j\overline{u^c}H^l\overline{H}_i\epsilon_{kl}$	$\frac{y_d y_u}{(16\pi^2)^3}\frac{v^2}{\Lambda}$	4×10^3	$\beta\beta0\nu$
28_b	$L^iL^jQ^kd^c\overline{Q}_k\overline{u^c}H^l\overline{H}_i\epsilon_{jl}$	$\frac{y_d y_u}{(16\pi^2)^3}\frac{v^2}{\Lambda}$	4×10^3	$\beta\beta0\nu$
28_c	$L^iL^jQ^kd^c\overline{Q}_l\overline{u^c}H^l\overline{H}_i\epsilon_{jk}$	$\frac{y_d y_u}{(16\pi^2)^3}\frac{v^2}{\Lambda}$	4×10^3	$\beta\beta0\nu$
29_a	$L^iL^jQ^ku^c\overline{Q}_k\overline{u^c}H^lH^m\epsilon_{il}\epsilon_{jm}$	$\frac{y_u^2}{(16\pi^2)^2}\frac{v^2}{\Lambda}\left(\frac{1}{16\pi^2}+\frac{v^2}{\Lambda^2}\right)$	2×10^5	$\beta\beta0\nu$
29_b	$L^iL^jQ^ku^c\overline{Q}_l\overline{u^c}H^lH^m\epsilon_{ik}\epsilon_{jm}$	$\frac{g^2}{(16\pi^2)^3}\frac{v^2}{\Lambda}$	4×10^4	$\beta\beta0\nu$
30_a	$L^iL^j\overline{L}_i\overline{e^c}\overline{Q}_k\overline{u^c}H^kH^l\epsilon_{jl}$	$\frac{y_\ell y_u}{(16\pi^2)^3}\frac{v^2}{\Lambda}$	2×10^3	$\beta\beta0\nu$
30_b	$L^iL^j\overline{L}_m\overline{e^c}\overline{Q}_n\overline{u^c}H^kH^l\epsilon_{ik}\epsilon_{jl}\epsilon^{mn}$	$\frac{y_\ell y_u}{(16\pi^2)^2}\frac{v^2}{\Lambda}\left(\frac{1}{16\pi^2}+\frac{v^2}{\Lambda^2}\right)$	2×10^3	$\beta\beta0\nu$
31_a	$L^iL^j\overline{Q}_i\overline{d^c}\overline{Q}_k\overline{u^c}H^kH^l\epsilon_{jl}$	$\frac{y_d y_u}{(16\pi^2)^2}\frac{v^2}{\Lambda}\left(\frac{1}{16\pi^2}+\frac{v^2}{\Lambda^2}\right)$	4×10^3	$\beta\beta0\nu$

Assumptions:

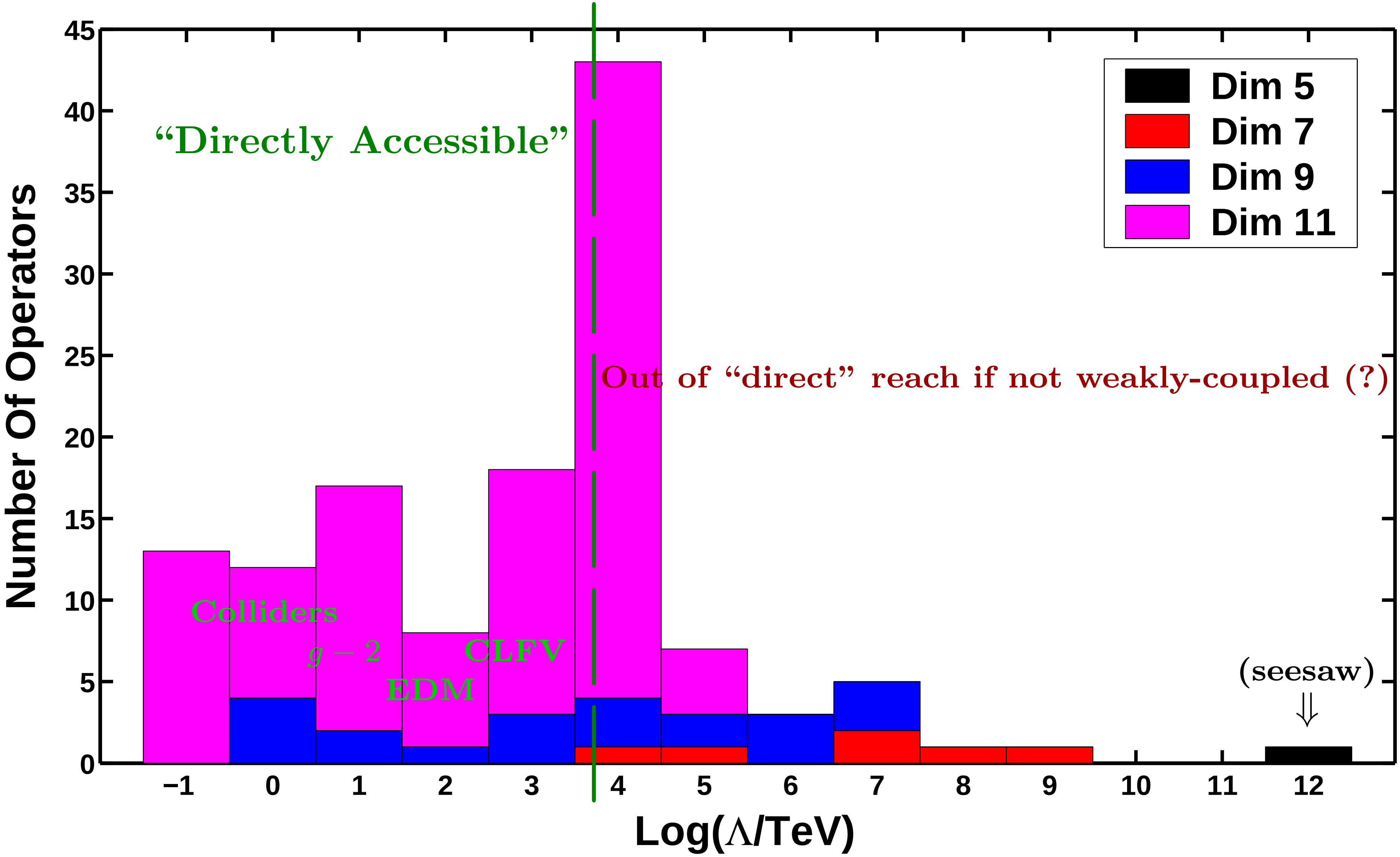
- Only consider $\Delta L = 2$ operators;
- Operators made up of only standard model fermions and the Higgs doublet (no gauge bosons);
- Electroweak symmetry breaking characterized by SM Higgs doublet field;
- Effective operator couplings assumed to be "flavor indifferent";
- Operators "turned on" one at a time, assumed to be leading order (tree-level) contribution of new lepton number violating physics.
- We can use the effective operator to estimate the coefficient of all other lepton-number violating lower-dimensional effective operators (loop effects, computed with a hard cutoff).

All results presented are order of magnitude *estimates*, <u>not</u> precise quantitative results.


(a)
LNV
Operator
ν_α
$\bar{\nu}_\beta$
(b)
yv
yv
ν_α
$\bar{\nu}_\beta$
(c)
yv
yv
y
H^-
$\bar{e}$
ν_α
y_β
$\bar{\nu}_\beta$
γ, g
W, Z
ν_α
h_0
h_0
$\bar{\nu}_\beta$
v
v
(d)
γ, g
W, Z
H^+
y
y
H^-
e
$\bar{e}$
ν_α
y_α
h_0
h_0
y_β
$\bar{\nu}_\beta$
v
v
(e)

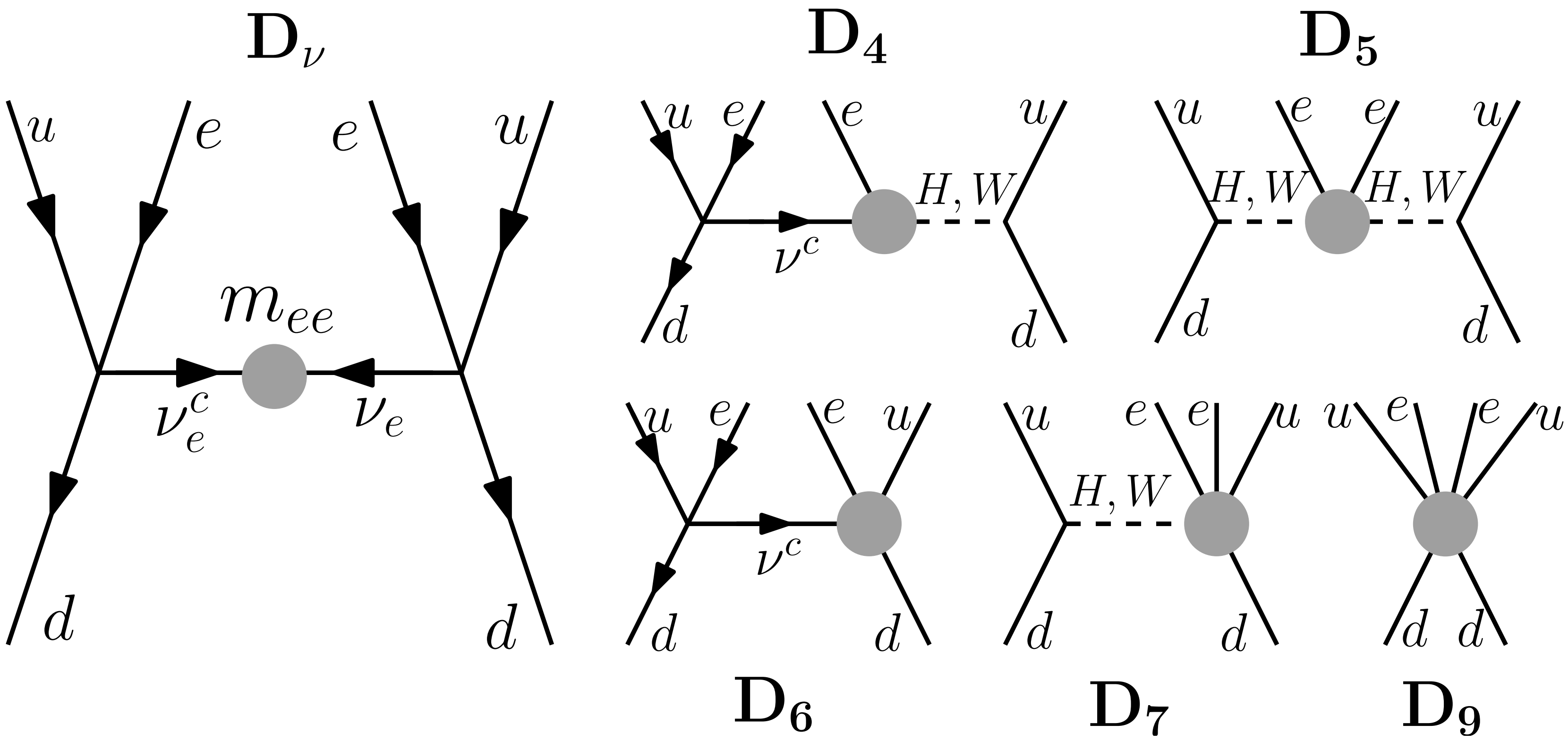


Number Of Operators
Log(Λ/TeV)
Dim 5
Dim 7
Dim 9
Dim 11
45
40
35
30
25
20
15
10
5
0
−1
0
1
2
3
4
5
6
7
8
9
10
11
12

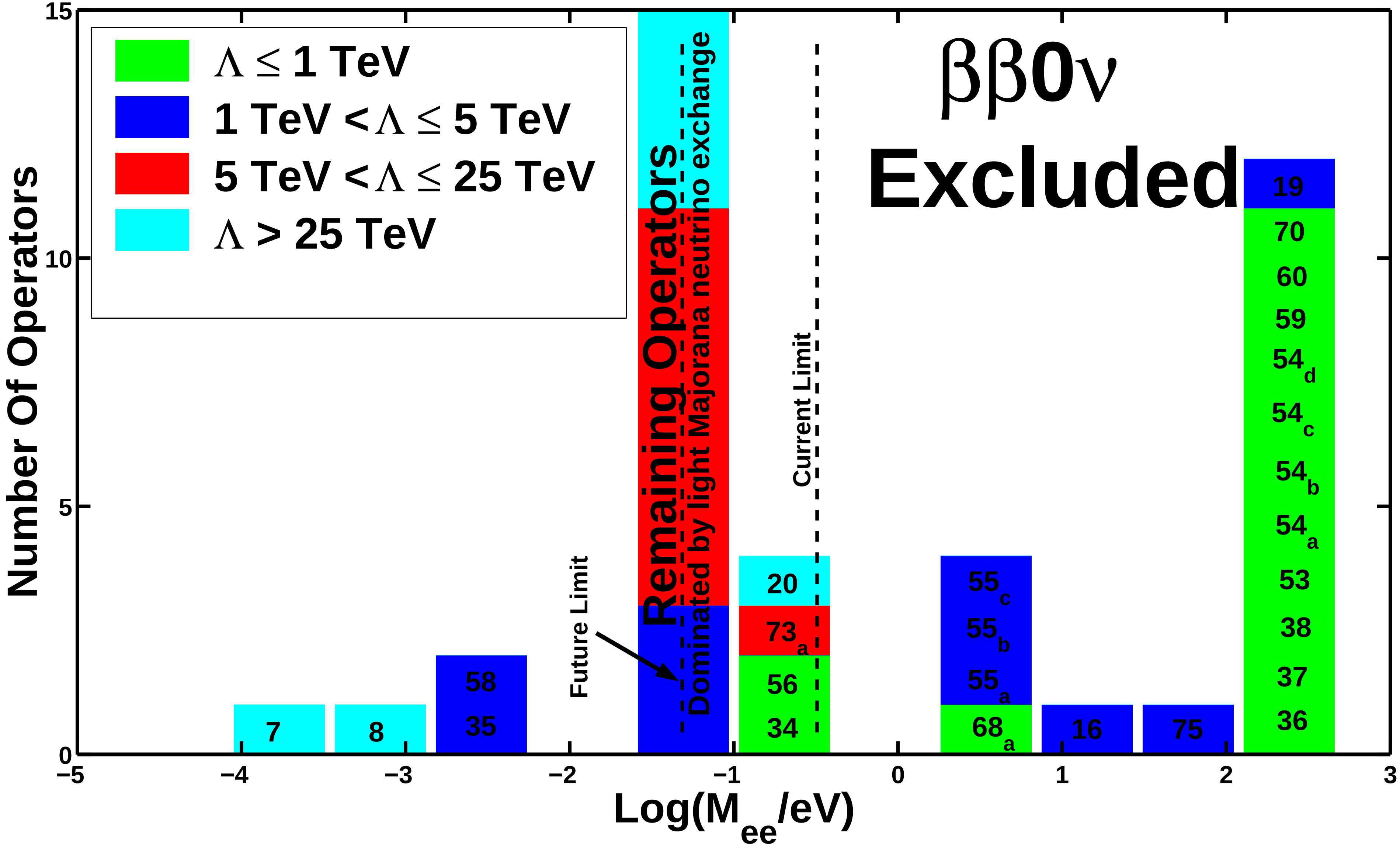


Dim 5
Dim 7
Dim 9
Dim 11
“Directly Accessible”
Out of “direct” reach if not weakly-coupled (?)
Colliders
$g-2$
EDM
CLFV
(seesaw)
Number Of Operators
Log(Λ/TeV)

Other Experimental Consequences: LNV Observables

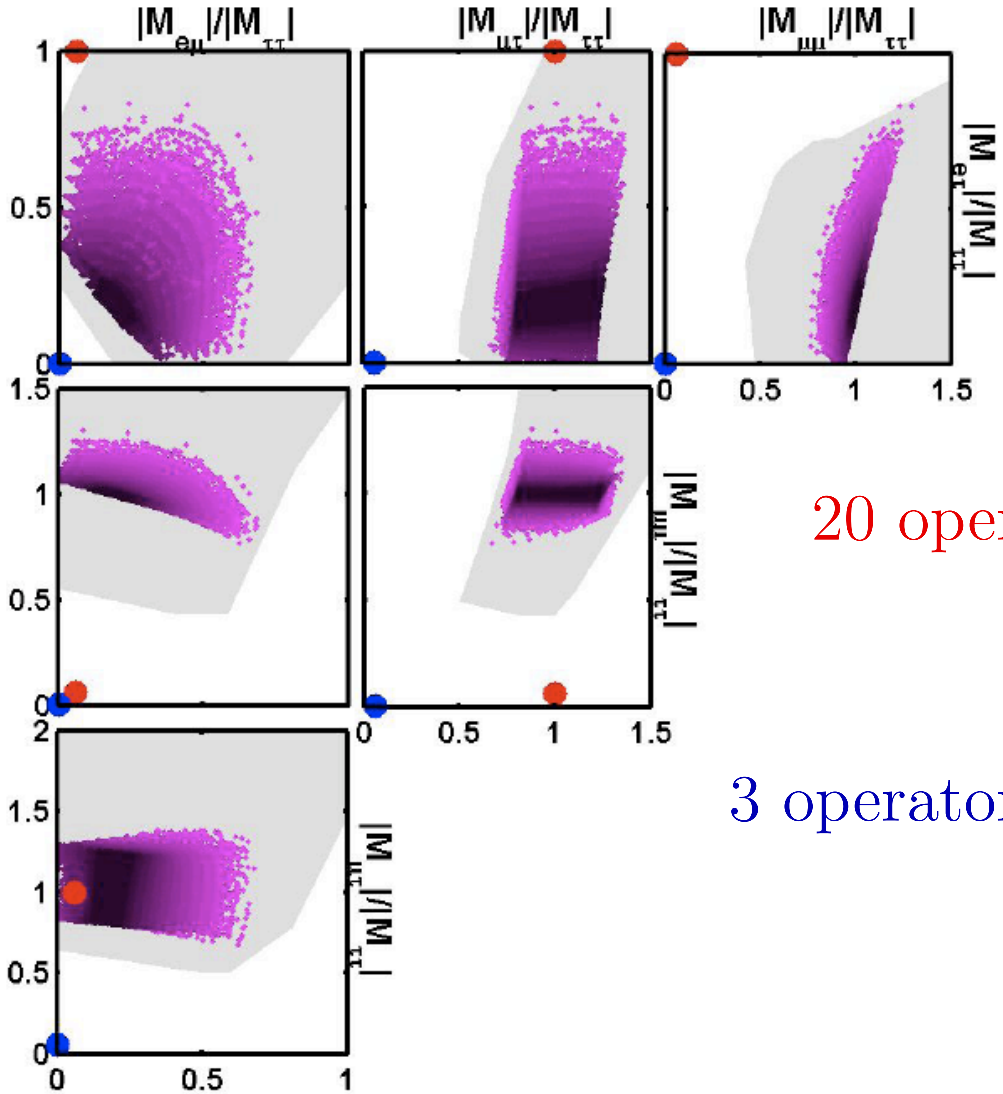


Neutrinoless Double-beta Decay ($0\nu\beta\beta$)



Λ ≤ 1 TeV
1 TeV < Λ ≤ 5 TeV
5 TeV < Λ ≤ 25 TeV
Λ > 25 TeV
ββ0ν
Excluded
Remaining Operators
Dominated by light Majorana neutrino exchange
Current Limit
Future Limit
Number Of Operators
Log(M_ee/eV)
7
8
58
35
20
73_a
56
34
55_c
55_b
55_a
68_a
16
75
19
70
60
59
54_d
54_c
54_b
54_a
53
38
37
36

Implied neutrino mass textures (numerical results)



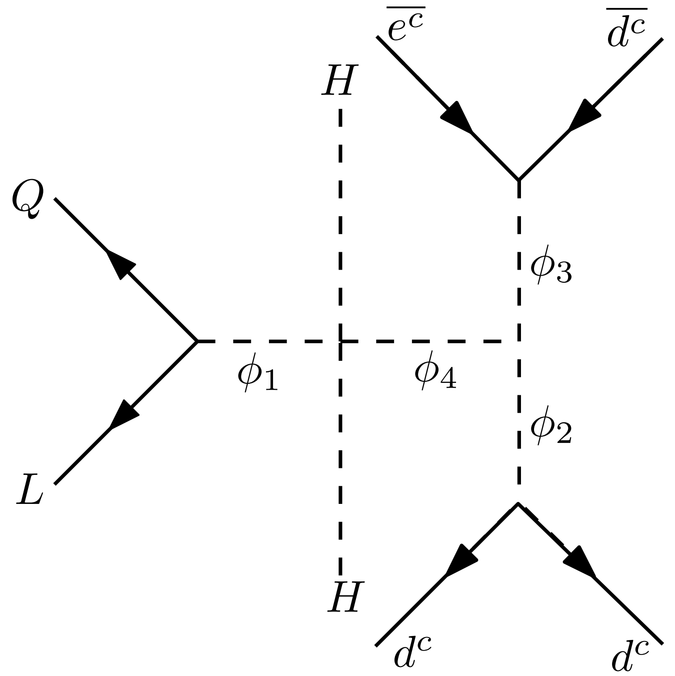
- Require $m_{ee} < 10^{-4}$ eV
- all phases vary freely
- 95% confidence limits
- θ_{13} varies from $0^\circ - 14^\circ$

20 operators: $M \propto \begin{pmatrix} \lambda_e & \lambda_\mu & \lambda_\tau \\ \lambda_\mu & \lambda_\mu & \lambda_\tau \\ \lambda_\tau & \lambda_\tau & \lambda_\tau \end{pmatrix}$

3 operators: $M \propto \begin{pmatrix} \lambda_e\lambda_e & \lambda_e\lambda_\mu & \lambda_e\lambda_\tau \\ \lambda_e\lambda_\mu & \lambda_\mu\lambda_\mu & \lambda_\mu\lambda_\tau \\ \lambda_e\lambda_\tau & \lambda_\mu\lambda_\tau & \lambda_\tau\lambda_\tau \end{pmatrix}$

Rest are "anarchical"

[arXiv:0708.1344 [hep-ph]]



Order-One Coupled, Weak Scale Physics Can Also Explain Naturally Small Majorana Neutrino Masses:

Multi-loop neutrino masses from lepton number violating new physics.

$$-\mathcal{L}_{\nu\rm SM} \supset \sum_{i=1}^{4} M_i\phi_i\bar{\phi}_i + iy_1 QL\phi_1 + y_2 d^c d^c\phi_2 + y_3 e^c d^c \phi_3 + \lambda_{14}\bar{\phi}_1\phi_4 HH + \lambda_{234}M\phi_2\bar{\phi}_3\phi_4 + h.c.$$

$m_\nu \propto (y_1 y_2 y_3 \lambda_{234})\lambda_{14}/(16\pi)^4 \;\rightarrow$ neutrino masses at 4 loops, requires $M_i \sim 100$ GeV!

WARNING: For illustrative purposes only. Details still to be worked out. Scenario most likely ruled out by charged-lepton flavor-violation, LEP, Tevatron, and HERA.

Understanding Fermion Mixing

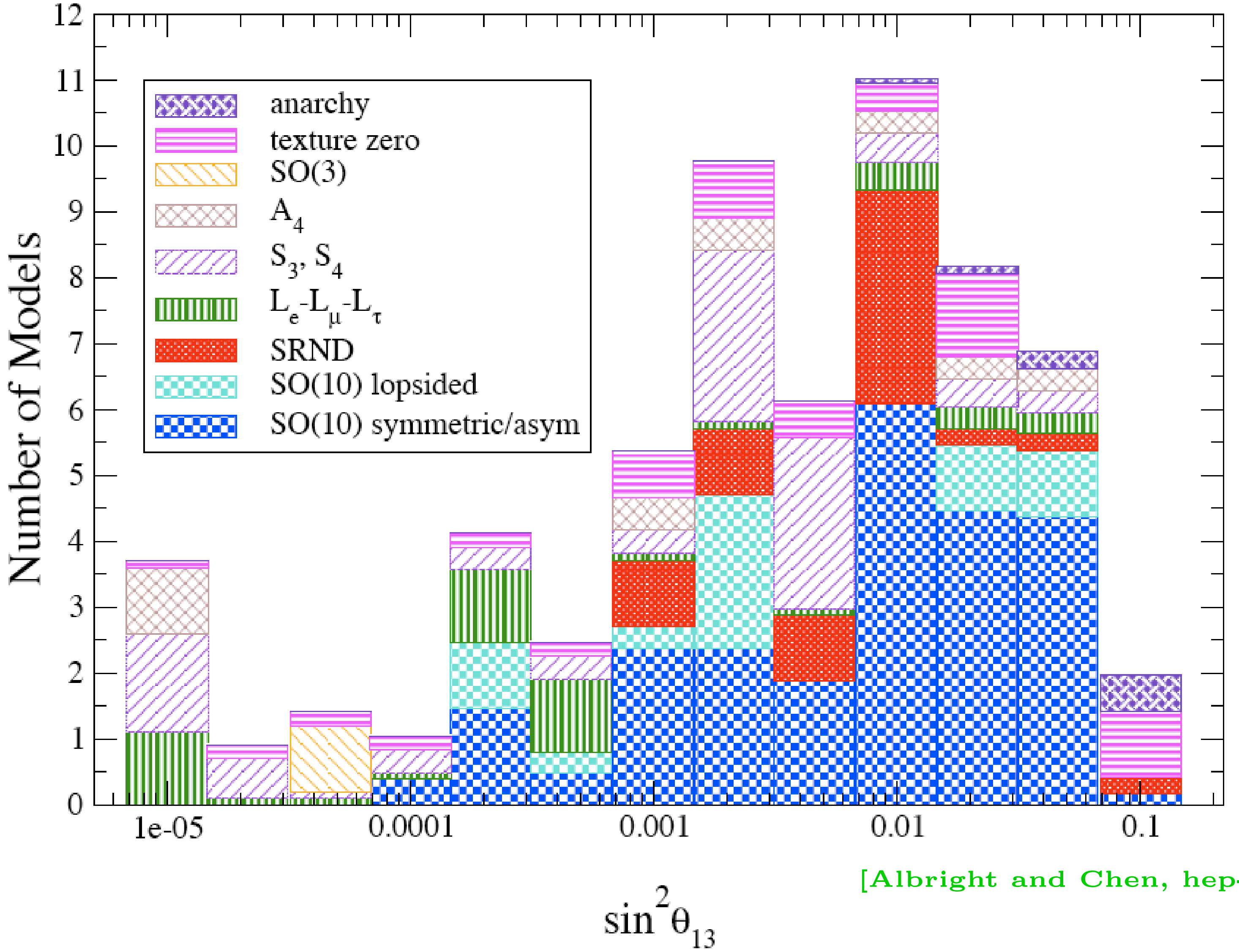
The other puzzling phenomenon uncovered by the neutrino data is the fact that Neutrino Mixing is Strange. What does this mean?

It means that lepton mixing is very different from quark mixing:

$$V_{MNS} \sim \begin{pmatrix} 0.8 & 0.5 & \mathbf{0.2} \\ 0.4 & 0.6 & 0.7 \\ 0.4 & 0.6 & 0.7 \end{pmatrix} \qquad V_{CKM} \sim \begin{pmatrix} 1 & 0.2 & 0.001 \\ 0.2 & 1 & 0.01 \\ 0.001 & 0.01 & 1 \end{pmatrix} \qquad \boxed{\text{WHY?}}$$

$[|(V_{MNS})_{e3}| < 0.2]$

They certainly look VERY different, but which one would you label as "strange"?



[Albright and Chen, hep-ph/0608137]

pessimist – "We can't compute what $|U_{e3}|$ is – must measure it!"

(same goes for the mass hierarchy, δ)

Comments On Current Flavor Model-Building Scene:

- VERY active research area. Opportunity to make *bona fide* prediction regarding parameters that haven't been measured yet but will be measured for sure in the near future $\rightarrow \theta_{13}$, δ, mass hierarchy, etc;

- For flavor symmetries, more important than determining the values of the parameters is the prospect of establishing non-trivial relationships among several interesting unkowns;

  e.g.,

  $$\sin^2\theta_{13} \sim \Delta m^2_{12}/|\Delta m^2_{13}| \text{ if hierarchy is normal,}$$
  $$\sin^2\theta_{13} \sim (\Delta m^2_{12}/|\Delta m^2_{13}|)^2 \text{ if hierachy is inverted}$$

  is common "prediction" of many flavor models (often also related to $\cos 2\theta_{23}$).

How Do We Learn More?

In order to learn more, we need more information. Any new data and/or idea is welcome, including

- searches for charged lepton flavor violation;

  ($\mu \to e\gamma$, $\mu \to e$-conversion in nuclei, etc)

- searches for lepton number violation;

  (neutrinoless double beta decay, etc)

- neutrino oscillation experiments;

  (Daya Bay, NOνA, etc)

- searches for fermion electric/magnetic dipole moments

  (electron edm, muon $g-2$, etc);

- precision studies of neutrino – matter interactions;

  (Minerνa, NuSOnG, etc)

- collider experiments:

  (LHC, etc)

  – *Can* we "see" the physics responsible for neutrino masses at the LHC? – YES!

    Must we see it? – NO, but we won't find out until we try!

  – we need to understand the physics at the TeV scale before we can really understand the physics behind neutrino masses (is there low-energy SUSY?, etc).

CONCLUSIONS

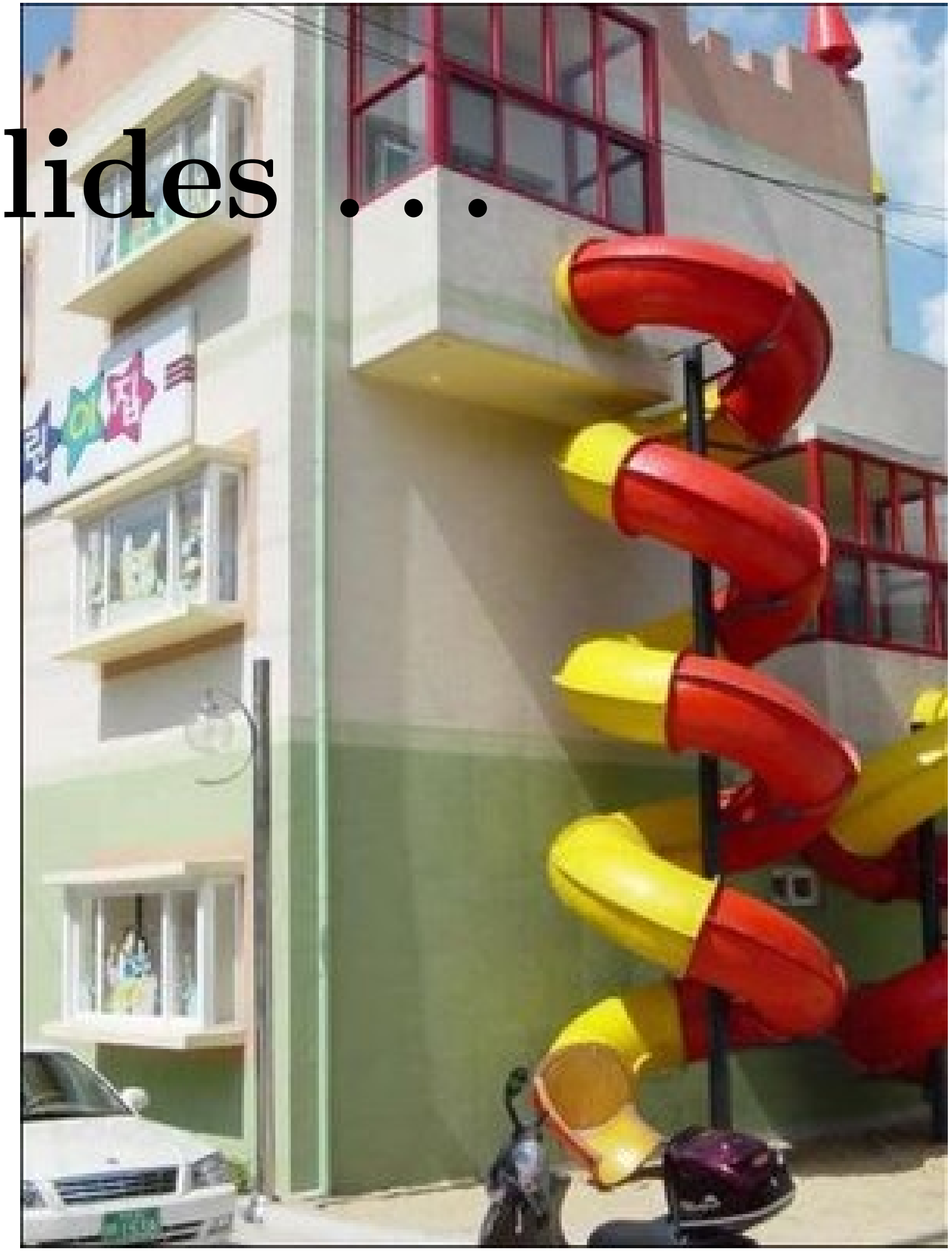
1. we have a very successful parametrization of the neutrino sector, but we still don't understand where neutrino masses (and lepton mixing) come from;

2. neutrino masses are very small – we don't know why, but we think it means something important;

3. we need a minimal νSM Lagrangian. In order to decide which one is "correct" we must uncover the faith of baryon number minus lepton number ($0\nu\beta\beta$ is the best bet? Likely, but not guaranteed).

4. We know very little about the new physics uncovered by neutrino oscillations.
   - It could be renormalizable $\rightarrow$ "boring" Dirac neutrinos
   - It could be due to Physics at absurdly high energy scales $M \gg 1$ TeV $\rightarrow$ high energy seesaw. How can we ever convince ourselves that this is correct?
   - It could be due to very light new physics $\rightarrow$ low energy seesaw. Prediction: new light propagating degrees of freedom – sterile neutrinos
   - It could be due to new physics at the TeV scale $\rightarrow$ either weakly coupled, or via a more subtle lepton number breaking sector. Predictions: charged lepton flavor violation, collider signatures!
5. We need more experimental data in order to decide what is really going on!

Backup Slides . . .



Another νSM

Why don't we just enhance the fermion sector of the theory?

One may argue that it is trivial and simpler to just add

$$\mathcal{L}_{\rm Yukawa} = -y_{i\alpha} L^i H N^\alpha + H.c.,$$

and neutrinos get a mass like all other fermions: $m_{i\alpha} = y_{i\alpha} v$

- Data requires $y < 10^{-12}$. Why so small?
- Neutrinos are Dirac fermions. $B - L$ exactly conserved.
- νSM is a renormalizable theory.

This proposal, however, violates the rules of the SM (as I defined them)! The operator $\frac{M_N}{2} NN$, allowed by all gauge symmetries, is absent. In order to explain this, we are forced to add a symmetry to the νSM. The simplest candidate is a global $U(1)_{B-L}$.

$U(1)_{B-L}$ is upgraded from accidental to fundamental (global) symmetry.

Old Standard Model, Encore

The SM is a quantum field theory with the following defining characteristics:

- Gauge Group $(SU(3)_c \times SU(2)_{\rm L} \times U(1)_{\rm Y})$;
- Particle Content (fermions: Q, u, d, L, e, scalars: H).

Once this is specified, the SM is unambiguously determined:

- Most General Renormalizable Lagrangian;
- Measure All Free Parameters, and You Are Done.

This model has *accidental global symmetries*. In particular, the anomaly free global symmetry is preserved: $U(1)_{B-L}$.

New Standard Model, Dirac Neutrinos

The SM is a quantum field theory with the following defining characteristics:

- Gauge Group ($SU(3)_c \times SU(2)_{\rm L} \times U(1)_{\rm Y}$);
- Particle Content (fermions: Q, u, d, L, e, N, scalars: H);
- Global Symmetry $U(1)_{B-L}$.

Once this is specified, the SM is unambiguously determined:

- Most General Renormalizable Lagrangian;
- Measure All Free Parameters, and You Are Done.

Naively not too different, but nonetheless qualitatively different $\rightarrow$ enhanced symmetry sector!

On very small Yukawa couplings

We would like to believe that Yukawa couplings should naturally be of order one.

Nature, on the other hand, seems to have a funny way of showing this. Of all known fermions, only one (1) has a "natural" Yukawa coupling – the top quark!

Regardless there are several very different ways of obtaining "naturally" very small Yukawa couplings. They require more new physics.

"Natural" solutions include flavor symmetries, extra-dimensions of different "warping," ...

Dirac Neutrinos and New Physics at the TeV Scale

If neutrinos are Dirac fermions, $M \equiv 0$, $\lambda < 10^{-12}$. Why so small?

Symmetry of the Lagrangian enhanced. Which symmetry? $\Rightarrow$ Lepton Number (more precisely $B - L$).

Another approach: enhanced SM gauge group, and choose charges such that the neutrino Yukawa coupling is not allowed (at a renormalizable level. . .)

Example: Neutrinos and a $U(1)_\nu$ Gauge Symmetry at the Weak Scale

Chen, AdG, Dobrescu hep-ph/0612017

Add to the SM $SU(3)\times SU(2)\times U(1)$ some gauge singlet fields (N right-handed neutrinos), and a new gauged $U(1)$ symmetry – $U(1)_\nu$. Add a new scalar sector responsible for spontaneous $U(1)_\nu$-breaking.

All SM fields are charged under $U(1)_\nu$ $(z_{q_i}, z_{u_i}, z_{d_i}, z_{\ell_i}, z_{e_i}, z_{n_i}, z_H)$.

We assume that for the different quark fields, $U(1)_\nu$ charges z_q, z_u, z_d are generation independent – this avoids pesky flavor-changing neutral currents in the quark sector.

We assume that $U(1)_\nu$ breaking occurs thorough the vev of a field ϕ with $z_\phi \equiv +1$.

Most importantly, we required the $SU(3)\times SU(2)\times U(1)\times U(1)_\nu$ gauge symmetry to be strictly anomaly free.

Other constraints

- quark Yukawa couplings are allowed at dimension 4 (top quark mass heavy);
- diagonal muon and tau Yukawa couplings allowed at dimension 4 (so that muon and tau necessarily get a mass).

Bottom-line: the presence of the right-handed neutrinos, plus the possibility that charges are generation dependent in the lepton sector allows non-trivial (i.e. different from hyper-charge and $B-L$) non-anomalous $U(1)_\nu$.

Neutrino Masses

$$\mathcal{L}_\nu = \sum_{i,j} \frac{c_\ell^{ij}}{\Lambda}\left(\frac{g_\phi}{\Lambda}\phi\right)^{q_{ij}} \overline{\ell^c}^i_L \ell^j_L H H + \sum_{i,k} \lambda_\nu^{ik}\left(\frac{g_\phi}{\Lambda}\phi\right)^{p_{ik}} \bar{\ell}^i_L n^k_R \tilde{H}$$
$$+ \sum_{k,k'} c_n^{kk'} \Lambda \left(\frac{g_\phi}{\Lambda}\phi\right)^{r_{kk'}} \overline{n^c}^k_R n^{k'}_R + \text{H.c.}\ .$$

Terms are only present when the exponents are integers. These are functions of the $U(1)_\nu$ particle charges:

$$\begin{aligned} p_{ik} &= z_u - z_q + z_{\ell_i} - z_{n_k}\ , \\ q_{ij} &= 2(z_q - z_u) - z_{\ell_i} - z_{\ell_j}\ , \\ r_{kk'} &= -z_{n_k} - z_{n_{k'}}\ . \end{aligned}$$

After spontaneous symmetry breaking

where

$$\epsilon \equiv g_\phi \frac{\langle\phi\rangle}{\Lambda} \ .$$

Remarkably, one can find commensurate, generation dependent lepton charges that cancel all gauge anomalies and satisfy the requirements listed earlier $\Rightarrow$ "Leptocratic Model".

More remarkable, perhaps, is the fact that we can obtain solutions that lead to naturally small neutrino masses – either Majorana or Dirac – the right pattern of lepton mixing, plus new light very weakly coupled ("sterile") fermion states.

Example: Orwellian Leptocratic Model

field	$U(1)_\nu$ charge
q_L	z_q
u_R	$4z_q + \frac{b}{3}$
d_R	$-2z_q - \frac{b}{3}$
ℓ_L	$-3z_q$
e_R	$-6z_q - \frac{b}{3}$
n_R^1	$-\frac{5b}{3}$
n_R^2, n_R^3	$\frac{4b}{3}$
H	$3z_q + \frac{b}{3}$
ϕ	$+1$

$$M_D = v\,\epsilon^{|b|}\begin{pmatrix} \lambda_\nu^{11}\,\epsilon^{|b|} & \lambda_\nu^{12} & \lambda_\nu^{13} \\ \lambda_\nu^{21}\,\epsilon^{|b|} & \lambda_\nu^{22} & \lambda_\nu^{23} \\ \lambda_\nu^{31}\,\epsilon^{|b|} & \lambda_\nu^{32} & \lambda_\nu^{33} \end{pmatrix} \quad ,$$

$$M_L = \frac{v^2}{\Lambda}\,\epsilon^{2|b|/3}\; c_\ell \quad ,$$

$$M_R = \Lambda\,\epsilon^{|b|/3}\begin{pmatrix} c_n^{11}\epsilon^{3|b|} & c_n^{12} & c_n^{13} \\ c_n^{12} & c_n^{22}\epsilon^{7|b|/3} & c_n^{23}\epsilon^{7|b|/3} \\ c_n^{13} & c_n^{23}\epsilon^{7|b|/3} & c_n^{33}\epsilon^{7|b|/3} \end{pmatrix} \quad .$$

If $|b|$ is not a multiple of 3, neutrinos are Dirac fermions. For $\lambda \sim 1$ and $|b| = 13$, two Dirac neutrinos weigh around 10^{-1} eV, while the third one weighs 10^{-14} eV. Mixing is "anarchical."

If, on the other hand, $|b| = 18$, $\Lambda \sim 1$ TeV, and $\epsilon = 0.1$, neutrinos are Majorana fermions (with the appropriate mass-squared differences and active–acitve mixing angles) and there are three other mostly sterile neutrinos with masses $m_{s2} \sim m_{s3} \sim 1$ MeV and $m_{s1} \sim 10^{-12}$ eV.

Active–sterile mixing is well-defined:

$$\Theta_{\rm active-heavy} \sim \epsilon^{12}\frac{v}{\Lambda} \sim 10^{-13} \ ,$$

$$\Theta_{\rm active-light} \sim \epsilon^{6}\frac{\Lambda}{v} \sim 10^{-5} \ .$$

Brief comment on Collider Phenomenology:

Associated to the spontaneously broken $U(1)_\nu$, there is a weak-scale Z' which is likely to be produced in collider experiments. The same is true of the $U(1)_\nu$ breaking sector (ϕ).

Salient features of the Z':

- mixes with the Z-boson if $z_H \neq 0$;
- non-universal coupling to different charged lepton families;
- a potentially very-large invisible width, since it also couples to several light "sterile" neutrinos.

Ratios of Z' branching ratios proportional to ratios of $U(1)_\nu$ charges: possible to experimentally verify flavor structure!

Predictions: **Tritium beta-decay**

Heavy neutrinos participate in tritium β-decay. Their contribution can be parameterized by

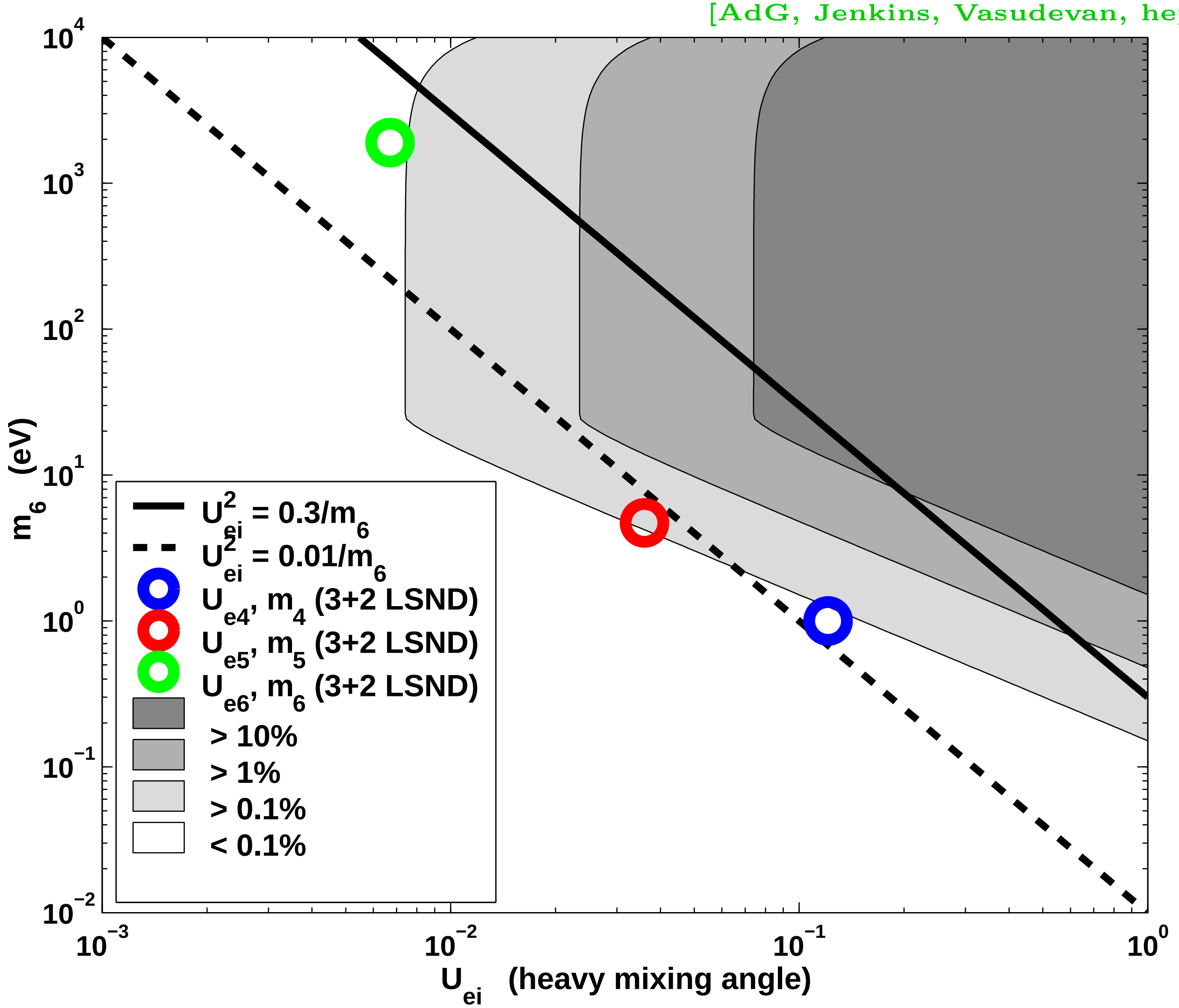
$$m_\beta^2 = \sum_{i=1}^{6} |U_{ei}|^2 m_i^2 \simeq \sum_{i=1}^{3} |U_{ei}|^2 m_i^2 + \sum_{i=1}^{3} |U_{ei}|^2 m_i M_i,$$

as long as M_i is not too heavy (above tens of eV). For example, in the case of a 3+2 solution to the LSND anomaly, the heaviest sterile state (with mass M_1) contributes the most: $m_\beta^2 \simeq 0.7\text{ eV}^2 \left(\frac{|U_{e1}|^2}{0.7}\right)\left(\frac{m_1}{0.1\text{ eV}}\right)\left(\frac{M_1}{10\text{ eV}}\right)$.

NOTE: next generation experiment (KATRIN) will be sensitive to $O(10^{-1})$ eV^2.

sensitivity of tritium beta decay to seesaw sterile neutrinos

[AdG, Jenkins, Vasudevan, hep-ph/0608147]



Brief Summary of Constraints:

Double-Beta Decay:

$$m_{ee}^{\rm heavy}=\sum_{i=1}^{n}(\lambda v)^2_{ei}M_i^{-3}Q^2.$$

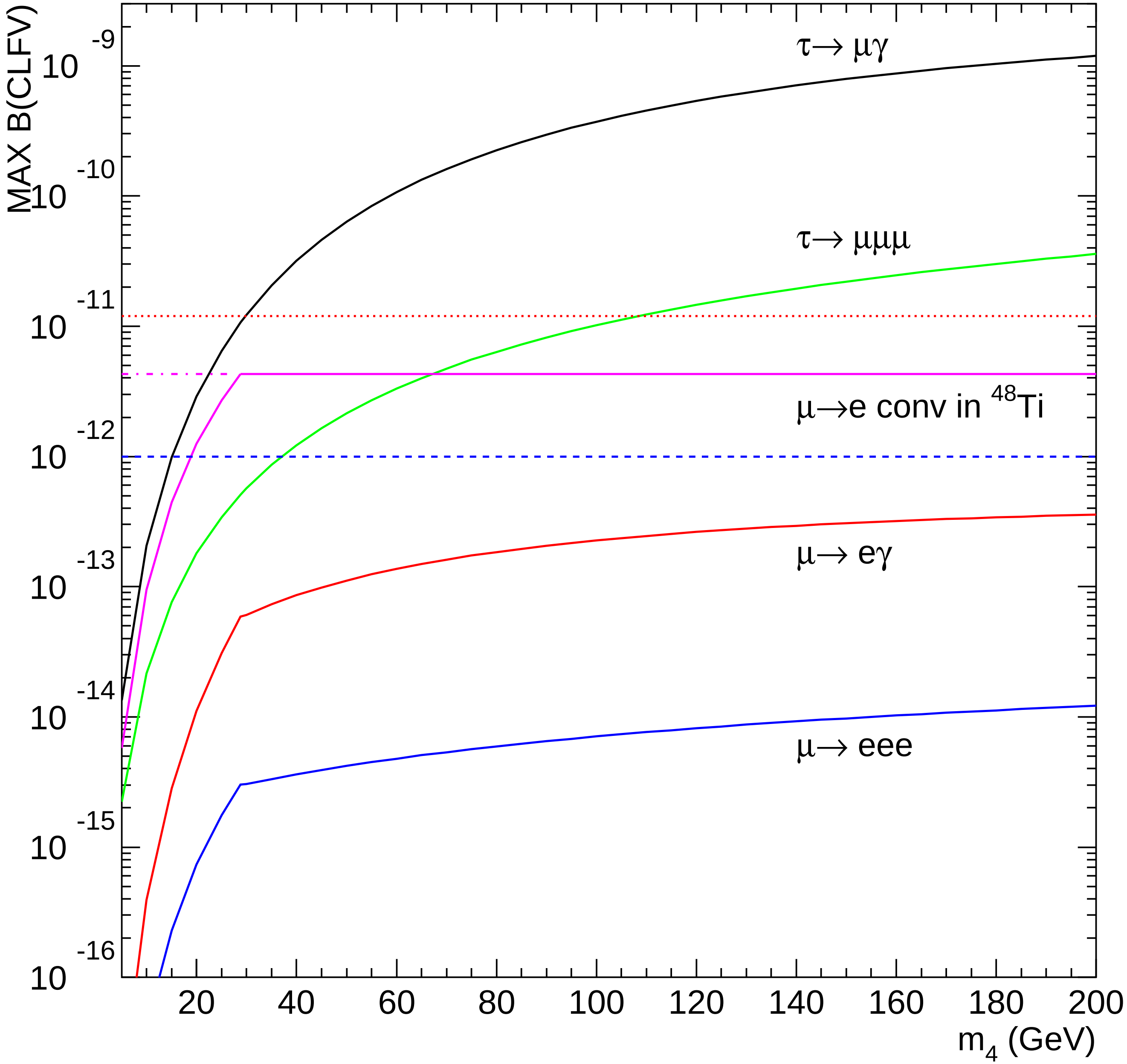
Universality tests:

$$\left|\sum_{k=4}^{3+n}\left(|U_{ek}|^2-|U_{\mu k}|^2\right)\right| < 0.004, \quad (\pi \text{ decay})$$

$$\left|\sum_{k=4}^{3+n}\left(|U_{ek}|^2-|U_{\tau k}|^2\right)\right| < 0.006, \quad (\tau \text{ decay})$$

$$\left|\sum_{k=4}^{3+n}\left(|U_{\mu k}|^2-|U_{\tau k}|^2\right)\right| < 0.006, \quad (\tau \text{ decay})$$

Constraint From Charged Lepton Flavor Violation



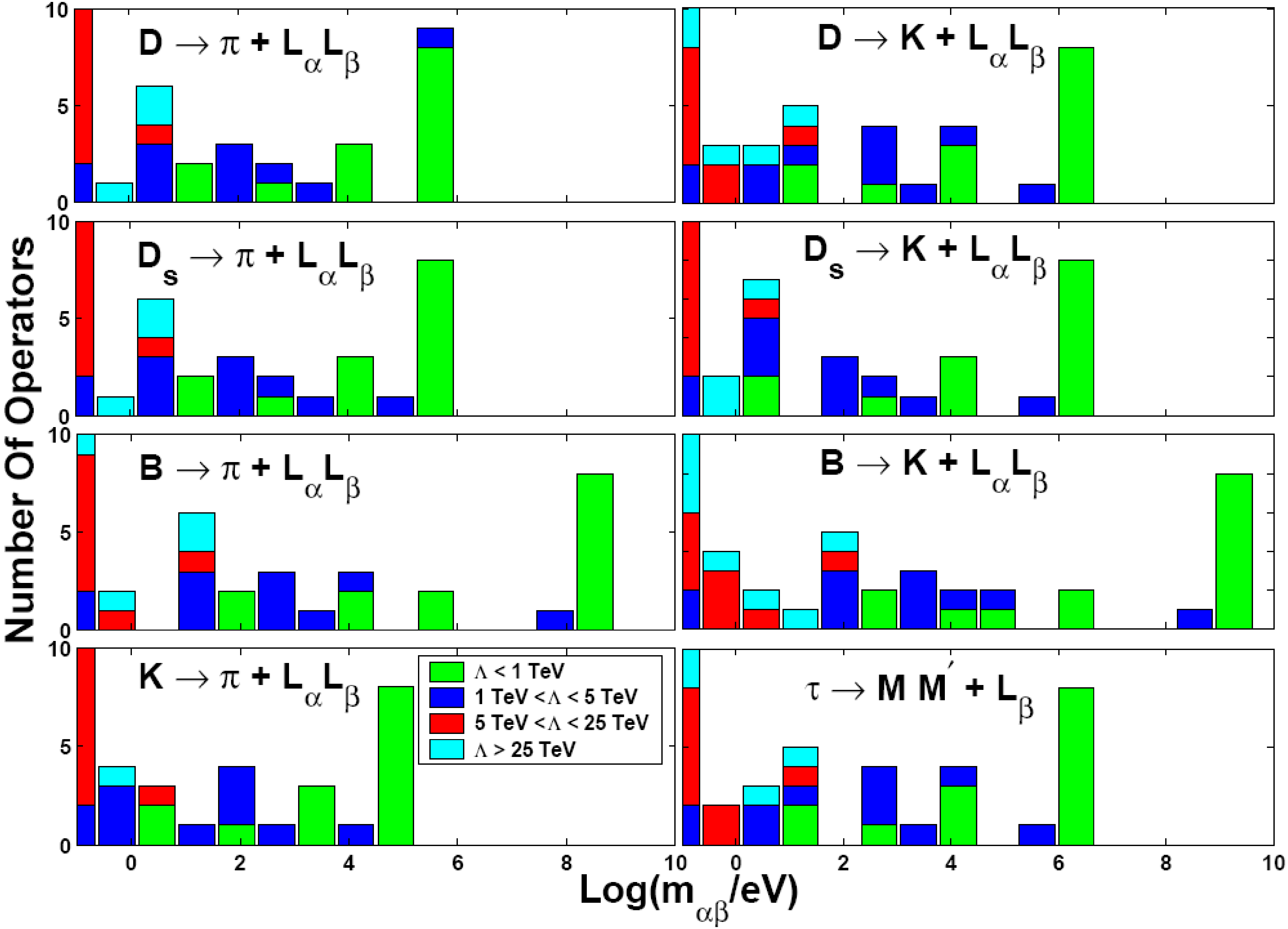
arXiv:0706.1732 [hep-ph]

Another stringent constraint: LEP

If right handed neutrino masses are below ~ 70 GeV, the flavor changing neutral current decay decay

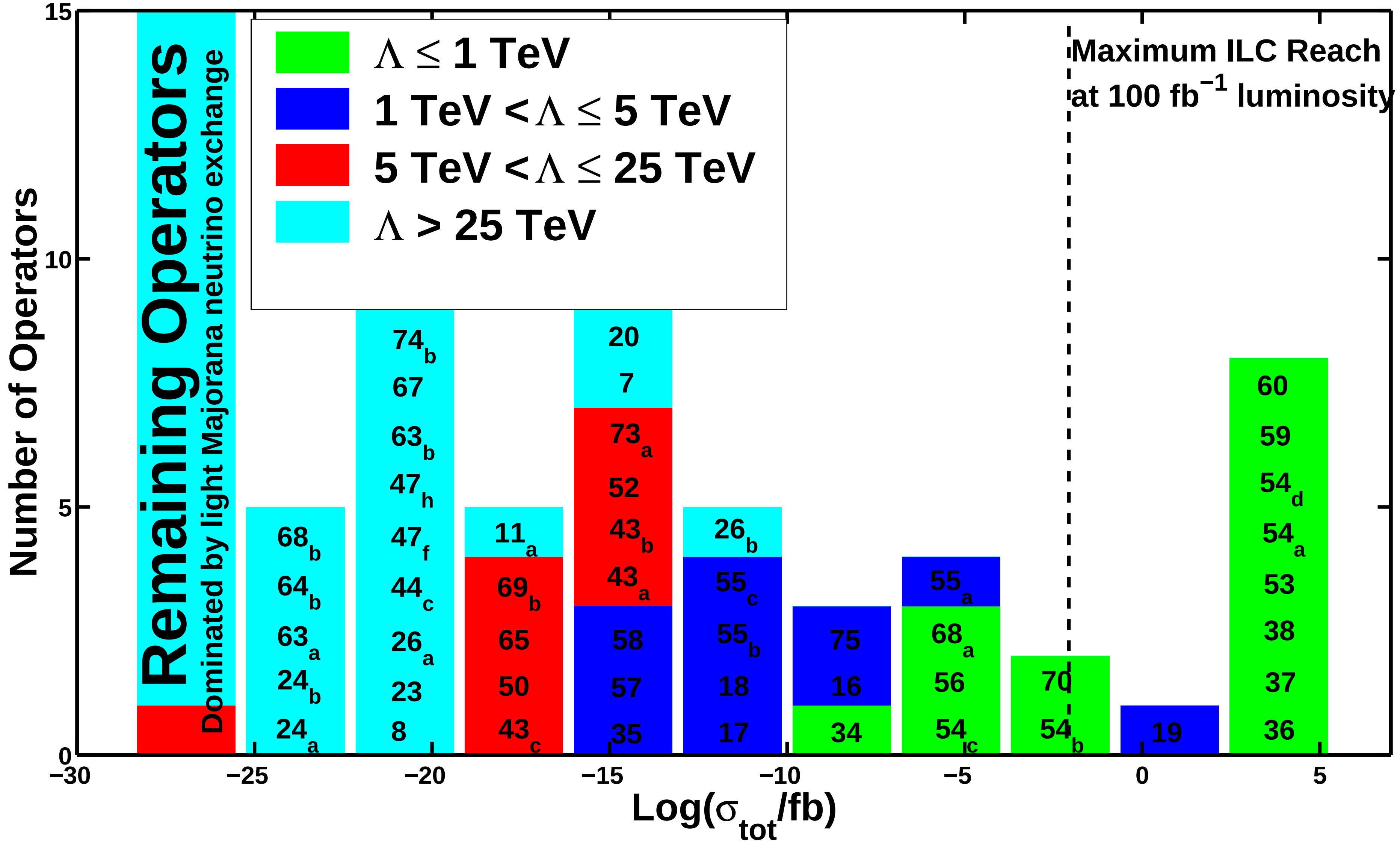
$$Z \to \nu\nu_4 \to \nu\ell W^*$$

Places the most severe constraints on $|U_{\alpha 4}|^2$ (α independent!)

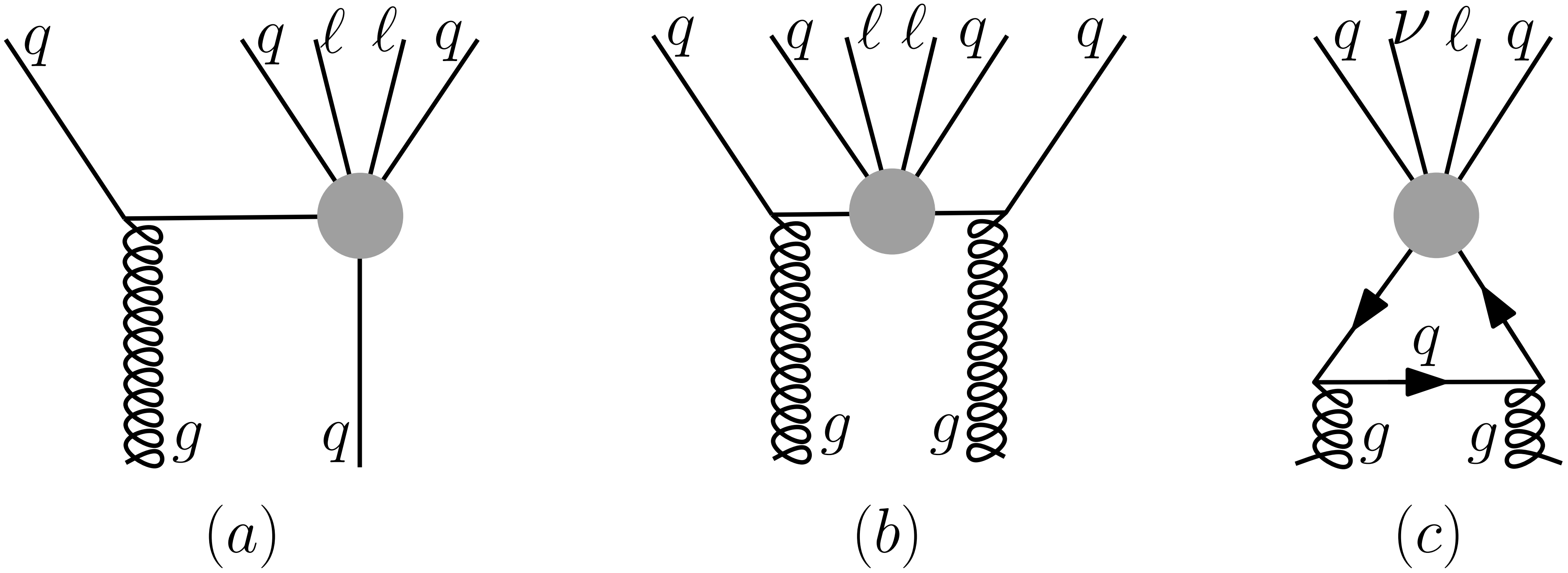


$D \rightarrow \pi + L_\alpha L_\beta$
$D \rightarrow K + L_\alpha L_\beta$
$D_s \rightarrow \pi + L_\alpha L_\beta$
$D_s \rightarrow K + L_\alpha L_\beta$
$B \rightarrow \pi + L_\alpha L_\beta$
$B \rightarrow K + L_\alpha L_\beta$
$K \rightarrow \pi + L_\alpha L_\beta$
$\tau \rightarrow M\, M' + L_\beta$
$\Lambda < 1$ TeV
1 TeV $< \Lambda <$ 5 TeV
5 TeV $< \Lambda <$ 25 TeV
$\Lambda >$ 25 TeV
Number Of Operators
$\mathrm{Log}(m_{\alpha\beta}/\mathrm{eV})$
0
2
4
6
8
10
5

LNV at Colliders ⇒1 TeV ILC: $e^-e^- \to 4$ jets, no missing energy



LNV at Colliders ⇒ LHC: $pp \to \ell^\pm\ell^\pm +$ multi-jets



OK — *OK* — ν in final state